1.) (a) correct substitution into the formula for the determinant (A1)
e.g. $\det \boldsymbol{A} = 9e^{x} \times e^{3x} - e^{x} \times e^{x}$

$\det \boldsymbol{A} = 9e^{4x} - e^{2x}$ A1 N2

(b) recognizing that no inverse implies $\det \boldsymbol{A} = 0$ R1
e.g. $9e^{4x} - e^{2x} = 0$, $ad - bc = 0$

attempt to solve equation (M1)
e.g. $e^{2x} = \frac{1}{9}$, $e^{-2x} = 9$, $e^{2x}(9e^{2x} - 1) = 0$, $9e^{4x} = e^{2x}$

rearranging to get correct log equation
e.g. $2x = \ln\frac{1}{9}, -2x = \ln 9, \ln(9e^{4x}) = \ln(e^{2x})$ (A1)

isolating x A1
e.g. $x\ \frac{1}{2}\ln\frac{1}{9}, x = -\frac{1}{2}\ln 9, x = \ln\frac{1}{3},\ a = -\frac{1}{2}, b = 9$
$x = -\ln 3$ (accept $a = -1$, $b = 3$) A1 N3

[7]

2.) (a) $\det \boldsymbol{M} = -4$ A1 N1

(b) $\boldsymbol{M}^{-1} = -\frac{1}{4}\begin{pmatrix} -1 & -1 \\ -2 & 2 \end{pmatrix} \left(= \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \right)$ A1A1 N2

Note: *Award A1 for* $-\frac{1}{4}$ *and A1 for the correct matrix.*

(c) $\boldsymbol{X} = \boldsymbol{M}^{-1}\begin{pmatrix} 4 \\ 8 \end{pmatrix} \left(\boldsymbol{X} = -\frac{1}{4}\begin{pmatrix} -1 & -1 \\ -2 & 2 \end{pmatrix}\begin{pmatrix} 4 \\ 8 \end{pmatrix} \right)$ M1

$\boldsymbol{X} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ $(x = 3, y = -2)$ A1A1 N0

Note: *Award no marks for an* ***algebraic*** *solution of the system 2x + y = 4, 2x - y = 8.*

[6]

3.) (a) $\boldsymbol{A}^{-1} = \begin{pmatrix} -1 & 1 & -1 \\ 1 & -0.5 & 1.25 \\ 1 & -0.5 & 0.75 \end{pmatrix}$ A2 N2

(b) (i) $\boldsymbol{I} - \frac{1}{2}\boldsymbol{B} = \boldsymbol{A}^{-1}$ A1

$-\frac{1}{2}\boldsymbol{B} = \boldsymbol{A}^{-1} - \boldsymbol{I}$ A1

$\boldsymbol{B} = -2(\boldsymbol{A}^{-1} - \boldsymbol{I})$ AG

(ii) $\boldsymbol{B} = \begin{pmatrix} 4 & -2 & 2 \\ -2 & 3 & -2.5 \\ -2 & 1 & 0.5 \end{pmatrix}$ A2 N2

(iii) det $\boldsymbol{B}$ = 12 A1 N1

(iv) det $\boldsymbol{B} \neq 0$ R1 N1

(c) (i) evidence of using a valid approach M1

e.g. $\boldsymbol{X} = \boldsymbol{B}^{-1}\boldsymbol{C}$

$\boldsymbol{X} = \begin{pmatrix} 0.333 \\ 1 \\ 1.33 \end{pmatrix} \left(= \begin{pmatrix} \frac{1}{3} \\ 1 \\ \frac{4}{3} \end{pmatrix} \right)$ A1 N1

(ii) $4x - 2y + 2z = 2$, $-2x + 3y - 2.5z = -1$, $-2x + y + 0.5z = 1$ A1A1A1 N3

[13]

4.) (a) $2\boldsymbol{A} = \begin{pmatrix} 6 & 4 \\ 2k & 8 \end{pmatrix}$ (A1)

$2\boldsymbol{A} - \boldsymbol{B} = \begin{pmatrix} 4 & 2 \\ 2k-1 & 5 \end{pmatrix}$ A2 N3

(b) Evidence of using the definition of determinant (M1)

Correct substitution (A1)

eg $4(5) - 2(2k - 1)$, $20 - 2(2k - 1)$, $20 - 4k + 2$

det $(2\boldsymbol{A} - \boldsymbol{B}) = 22 - 4k$ A1 N3

[6]

5.) $2p^2 + 12p = 14$ (M1) (A1)

$p^2 + 6p - 7 = 0$

$(p + 7)(p - 1) = 0$ (A1)

$p = -7$ or $p = 1$ (A1) (C4)

Note: *Both answers are required for the final (A1).*

[4]